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Generalized Landauer formulae for quasi-particle transport in disordered superconductors

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Abstract. When a quasi-particle current passes into a disordered superconductor, from ideal normal leads connected to external reservoirs, a finite electrical resistance R_S arises from scattering processes within the superconductor. A new formula for R_S is obtained, which reduces to the well-known Landauer formula in the absence of superconductivity. If $R_0, R_a(T_0, T_a)$ are reflection (transmission) coefficients associated with normal and Andreev scattering respectively, one finds, in one dimension at zero temperature, $R_S = (h/2e^2)(R_0 + T_a + \delta)/(R_a + T_0 - \delta)$ where δ is a small parameter arising from the absence of inversion symmetry. Generalizations of this result to finite temperatures and higher dimensions are also obtained.

Whereas, in certain circumstances, there is competition between localization and superconductivity [1–3], this need not always be the case. Recently [4, 5], it has been demonstrated that quasi-particle excitations in short coherence length superconductors can be localized by random spatial fluctuations in the order parameter $\Delta(\mathbf{r})$. This is a new mechanism for localization, since the disorder appears in the off-diagonal elements of the Bogoliubov–de Gennes (BG) equation, while for conventional Anderson localization the disorder is restricted to diagonal terms in the BG operator. Since order parameter fluctuations can lead to enhanced Andreev scattering, one expects measurements of the boundary resistance [6–9] between normal (N) and superconducting (S) materials to yield important information about disorder-induced transitions. In such experiments, a quasi-particle penetrates deep into the superconductor before being reflected, either as a particle or hole, back into the normal material. The boundary resistance arises because in the penetration region, where conversion from normal current to supercurrent occurs, there may be a finite probability for reflection without conversion.

In order to take advantage of measurements such as these, two items of theoretical machinery are needed. First, for a given model of disorder, techniques are needed for computing the various reflection and transmission coefficients of, for example, NS, NSN or SNS junctions. For simple junctions with no disorder, this problem has been analysed by several authors [10–13]. In the presence of disorder [4, 5], the problem can be tackled numerically by taking advantage of a range of optimized techniques [14–18] developed to describe localization in normal solids. The second item of theoretical machinery is a formula for computing the boundary resistance R_S from known reflection and transmission coefficients. Remarkably, no such formula presently exists. Early work on N–S boundaries, based on the Boltzmann equation

[7, 8] suggests that

$$R_S \sim l_a/l_0 \quad (1)$$

where $l_a(l_0)$ is the scattering length for Andreev (normal) processes within the superconductor. However such a formula, by its very nature, cannot adequately describe localization, where phase coherence can lead to substantial corrections to results obtained from Boltzmann-type arguments.

The aim of this letter is to derive a general formula for R_S in terms of reflection and transmission coefficients. For normal solids, the corresponding result is the Landauer formula [19–23], which has been used extensively during the past decade to investigate the effect of disorder on transport properties. In what follows a generalization of the Landauer approach is described. Initially, attention will be restricted to a zero temperature, one-dimensional superconductor in the interval $0 < x < L$ and later the results will be generalized to finite temperatures and higher dimensions. Since a disordered, one-dimensional superconductor cannot exist in nature, the one-dimensional analysis is aimed primarily at illustrating key ideas. It should be noted, however, that the results obtained in one dimension could be applied to simple models of layered materials with a current normal to the layers and a random layer thickness. In this case, the transverse k -vector of an incoming quasi-particle is conserved and the problem is equivalent to that of many one-dimensional channels, conducting in parallel.

Following Büttiker *et al* [23], long normal leads at chemical potentials μ_A and μ_B are attached to the left and right of the scattering region and these are in turn, attached to quasi-particle reservoirs at chemical potentials μ_1 and μ_2 (see figure 1). The leads are identical, perfect, one-dimensional conductors, whose chemical potentials must be chosen self-consistently to yield the correct electron density in the presence of a current I . In addition, in order to ensure quasi-particle charge conservation, it will be shown that the chemical potential μ of the superconductor must also be chosen self-consistently.

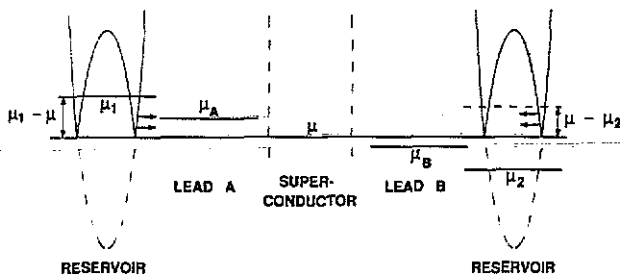


Figure 1. This shows a superconductor of chemical potential μ connected to perfect leads at chemical potentials μ_A and μ_B , which are themselves in contact with reservoirs at chemical potentials μ_1 and μ_2 . Parabolic, free electron energy bands in the reservoirs are filled up to μ_1 and μ_2 . The figure shows these reflected about the line $E = \mu$ to yield the quasi-particle picture appropriate to a superconductor. In this picture at zero temperature, all particle states in the left reservoir between μ_1 and μ are filled, as are all hole states, in the right reservoir between $(\mu - \mu_2) + \mu$ and μ .

The 4×4 S -matrix connecting incoming to outgoing quasi-particle amplitudes of a given energy E (measured relative to μ), obtained by solving the unperturbed BG

equation, can be written as follows.

$$S(E) = \begin{pmatrix} r & t \\ t' & r' \end{pmatrix} \tag{2}$$

where r, r' and t, t' are 2×2 reflection and transmission matrices for quasi-particle amplitudes. The corresponding matrix for the reflection and transmission coefficients $\rho(E)$ connecting incoming to outgoing fluxes, formed by taking the modulus square of the elements of S and multiplying by the ratio of outgoing to incoming group velocities, is of the form [4]

$$\rho(E) = \begin{pmatrix} R_{PP} R_{Ph} & T_{PP'} T_{Ph'} \\ R_{hP} R_{hh} & T_{hP'} T_{hh'} \\ T_{P'P} T_{P'h} & R_{P'P'} R_{P'h'} \\ T_{h'P} T_{h'h} & R_{h'P'} R_{h'h'} \end{pmatrix}. \tag{3}$$

For a unit particle flux incident from the left, R_{PP} and R_{hP} are the reflected particle and hole fluxes respectively and $T_{P'P}$ and $T_{h'P}$ are the transmitted particle and hole fluxes. Column 2 of ρ contains the corresponding outgoing fluxes arising from a unit incident hole flux from the left, while columns 3 and 4 yield outgoing fluxes associated with incident particles and holes from the right. In general, the only constraints on the 16 elements of ρ are those arising from the conservation of quasi-particle probability, which requires $\sum_{i=1}^4 \rho_{ij} = \sum_{j=1}^4 \rho_{ij} = 1$.

Following Büttiker *et al* [23], the reservoirs are taken to be incoherent sources of waves. At $T = 0$, with $\mu_1 > \mu_2$, particles with energies between μ and μ_1 are emitted from the left and holes with energies between μ and $\mu + (\mu - \mu_2)$ are emitted from the right. In the leads, the group velocities of particles and holes at $E = 0$ are equal to the Fermi velocity v_F . Hence the current for two spin directions in the left lead is

$$I = ev_F N(0) \{ (\mu_1 - \mu)(1 - R_{PP} + R_{hP}) + (\mu - \mu_2)(T_{hh'} - T_{Ph'}) \} \tag{4}$$

where $N(0)$ is the density of states per unit length in the leads for particles with positive velocity at $E = 0$. Since $v_F N(0) = 2/h$, the prefactor in equation (4) is independent of the characteristics of the leads.

To obtain an expression for the total resistance $R_S = (\mu_A - \mu_B)/(eI)$, expressions for the chemical potentials μ_A and μ_B are needed. At equilibrium, μ_A and μ_B are related to the excess charge density in the leads through the equations

$$2N(0)(\mu_A - \mu) = N(0) \{ (\mu_1 - \mu)(1 - R_{hP} + R_{PP}) + (\mu - \mu_2)(T_{Ph'} - T_{hh'}) \} \tag{5}$$

and

$$2N(0)(\mu_B - \mu) = N(0) \{ (\mu_1 - \mu)(T_{P'P} - T_{h'P}) + (\mu - \mu_2)(-1 - R_{h'h'} + R_{P'h'}) \} \tag{6}$$

where the factor of 2 on the left-hand sides arises because $N(0)$ is the density of states for one velocity direction only. In writing down the right-hand sides, it has been noted that the charge of a hole is equal and opposite to that of a particle. Combining these expressions yields

$$\mu_A - \mu_B = (\mu_1 - \mu)(R_{PP} + T_{h'P}) + (\mu - \mu_2)(R_{h'h'} + T_{Ph'}). \tag{7}$$

At this point one notes that, in the absence of particle-hole scattering where $T_{h'P} = T_{Ph'} = 0$ and $R_{PP} = R_{h'h'}$, equation (7) reduces to $\mu_A - \mu_B = R_{PP}(\mu_1 - \mu_2)$ and equation (4) to $I = (2e/h)T_{PP'}(\mu_1 - \mu_2)$. Hence the difference $\mu_1 - \mu_2$ can be eliminated to yield the well known Landauer formula [19, 20]: $R_S = (h/2e^2)(R_{PP}/T_{PP'})$. In the presence of particle-hole scattering, to eliminate μ from equations (4) and (7), an additional condition is required. To proceed further, it is crucial to recognize that the quasi-particle charge is not automatically a conserved quantity. Consider for example the effect of the incident particle current from the left. The difference between the incident and outgoing currents is

$$\begin{aligned}\delta i_{\text{left}} &= ev_F N(0)(\mu_1 - \mu)[1 - (R_{PP} - R_{hP} + T_{P'P} - T_{h'P})] \\ &= ev_F N(0)(\mu_1 - \mu)2(R_{hP} + T_{h'P}).\end{aligned}$$

Similarly for the hole current from the right

$$\delta i_{\text{right}} = ev_F N(0)(\mu - \mu_2)(-2)(R_{P'h'} + T_{Ph'}).$$

Since, at equilibrium, the charge on the superconductor remains constant, we require $\delta i_{\text{left}} + \delta i_{\text{right}} = 0$. Hence the equilibrium value of μ must satisfy $(R_{P'h'} + T_{Ph'}) (\mu - \mu_2) = (R_{hP} + T_{h'P})(\mu_1 - \mu)$. Combining this with equation (7) yields expressions for $\mu - \mu_2$ and $\mu_1 - \mu$ in terms of $\mu_A - \mu_B$ which, when inserted into equation (4), yields an expression for the dimensionless resistance $\bar{R} = R_S/(h/2e^2)$ of the form

$$\bar{R} = [R_{PP} + T_{h'P} + \alpha(R_{h'h'} + T_{Ph'})]/[1 - R_{PP} + R_{hP} + \alpha(T_{hh'} - T_{Ph'})] \quad (8a)$$

where $\alpha = (R_{hP} + T_{h'P})/(R_{P'h'} + T_{Ph'})$.

This general result is simplified at $T = 0$ where only states at $E = 0$ contribute. At this energy, particle-hole symmetry requires that the elements of $\rho(0)$ are invariant under an interchange of indices $P \leftrightarrow h$, $P' \leftrightarrow h'$. Furthermore, if the unperturbed system is invariant under time reversal, $\rho_{ij} = \rho_{ji}$. In the presence of these symmetries†, equation (8) is simplified by introducing average normal and Andreev reflection coefficients $R_0 = (R_{PP} + R_{P'P'})/2$ and $R_a = (R_{hP} + R_{h'P'})/2$. Introducing the corresponding transmission coefficients $T_0 = T_{PP'}$, $T_a = T_{hP}$, and a parameter $\delta = (R_{hP} - R_{h'P'})^2/2(R_{hP} + R_{h'P'} + 2T_a)$, which characterizes the spatial asymmetry of the system, yields

$$\bar{R} = \frac{R_0 + T_a + \delta}{R_a + T_0 - \delta} \quad (8b)$$

In general, $\delta \neq 0$ because a disordered system lacks inversion symmetry.

These expressions for \bar{R} reveal, in a transparent manner, several key properties of N-S-N junctions. In the absence of superconductivity, when the Andreev reflection and transmission coefficients R_a, T_a vanish, $\delta = 0$ and equation (8b) reduces to the Landauer formula in one dimension. For an infinite system with spatial fluctuations in the order parameter $\Delta(r)$ only, to quasi-classical accuracy, $R_a = 1$ and all other coefficients vanish [4]. Hence in this case, $\bar{R} = 0$. More generally, in the presence

† With these symmetries there are only four distinct reflection coefficients R_{PP} ($= R_{hh}$), R_{Ph} ($= R_{hP}$), $R_{P'P'}$ ($= R_{h'h}$) and $R_{P'h'}$ ($= R_{h'P'}$) and two distinct transmission coefficients $T_{P'P}$ ($= T_{P'P'}$ = $T_{hh'} = T_{h'h}$) and $T_{h'P}$ ($= T_{P'h} = T_{Ph'} = T_{h'P}$).

of both diagonal and off-diagonal disorder, the coefficients on the right-hand side of equations (8) must be evaluated through detailed calculations based on specific models. It is interesting to note that equation (8a) can be written in the form $\bar{R} = \bar{R}_{\text{left}} + \bar{R}_{\text{right}}$, where the left and right boundary resistances are defined to be $\bar{R}_{\text{right}} = (2e/h)(\mu_A - \mu)/I$ and $\bar{R}_{\text{left}} = (2e/h)(\mu - \mu_B)/I$. Using equations (4) to (6), these expressions are readily evaluated to yield

$$\bar{R}_{\text{left}} = \frac{1}{2} \frac{R_{\text{PP}} + a + b}{R_{\text{hP}} + a - b} \quad \text{and} \quad \bar{R}_{\text{right}} = \frac{1}{2} \frac{R_{\text{h'h'}} + a' + b'}{R_{\text{P'h'}} + a' - b'}$$

where $a = (T_{\text{P'P}} + T_{\text{h'P}})/2$, $b = (T_{\text{Ph'}} - T_{\text{hh'}})\alpha/2$ and $a' = (T_{\text{Ph'}} + T_{\text{hh'}})/2$, $b' = (T_{\text{h'P}} - T_{\text{P'P}})/2\alpha$. As expected, \bar{R}_{right} is obtained from \bar{R}_{left} by making the interchange of indices $P \leftrightarrow h', h \leftrightarrow P'$. These expressions are particularly useful in the long wire limit $L \rightarrow \infty$, where all transmission coefficients vanish. In this case, $\bar{R}_{\text{left}} = R_{\text{PP}}/2R_{\text{hP}}$ and $\bar{R}_{\text{right}} = R_{\text{h'h'}}/2R_{\text{P'h'}}$, which represent generalizations of equation (1) to the case where phase coherence is preserved during scattering processes within the superconductor.

This analysis is readily generalized to finite temperatures. Provided the temperature is much less than the Fermi temperature, small differences in the particle and hole group velocities can be ignored and the density of states can be taken outside the energy integrals. Since particles and holes are now emitted from both reservoirs writing $e\phi_1 = \mu_1 - \mu$, $e\phi_2 = \mu - \mu_2$, yields for the current

$$I = \frac{2e}{h} \int_0^\infty dE \{ f(E - e\phi_1)[1 - R_{\text{PP}} + R_{\text{hP}}] + f(E - e\phi_2)[T_{\text{hh'}} - T_{\text{Ph'}}] + f(E + e\phi_1)[-1 - R_{\text{Ph}} + R_{\text{hh}}] + f(E + e\phi_2)[T_{\text{hP'}} - T_{\text{PP'}}] \} \quad (9)$$

where f is the Fermi function. Similarly, writing $e\phi_A = \mu_A - \mu$ and $e\phi_B = \mu - \mu_B$, the requirements that μ_A and μ_B produce the correct equilibrium charge densities in the leads, yields

$$2 \int_0^\infty dE \{ f(E - e\phi_A) - f(E + e\phi_A) \} \\ = \int_0^\infty dE \{ f(E - e\phi_1)[1 + R_{\text{PP}} - R_{\text{hP}}] + f(E - e\phi_2)[-T_{\text{hh'}} + T_{\text{Ph'}}] + f(E + e\phi_1)[-1 + R_{\text{Ph}} - R_{\text{hh}}] + f(E + e\phi_2)[-T_{\text{hP'}} + T_{\text{PP'}}] \}$$

and

$$2 \int_0^\infty dE \{ f(E + e\phi_B) - f(E - e\phi_B) \} \\ = \int_0^\infty dE \{ f(E - e\phi_1)[T_{\text{P'P}} - T_{\text{h'P}}] + f(E - e\phi_2)[-1 + R_{\text{P'h'}} - R_{\text{h'h'}}] + f(E + e\phi_1)[T_{\text{P'h}} - T_{\text{h'h}}] + f(E + e\phi_2)[1 + R_{\text{P'P'}} - R_{\text{h'P'}}] \}.$$

Hence

$$\int_0^\infty dE \{ f(E - e\phi_A) + f(E - e\phi_B) - f(E + e\phi_A) - f(E + e\phi_B) \} \\ = \int_0^\infty dE \{ f(E - e\phi_1)[R_{\text{PP}} + T_{\text{h'P}}] + f(E - e\phi_2)[R_{\text{h'h'}} + T_{\text{Ph'}}] - f(E + e\phi_1)[R_{\text{hh}} + T_{\text{P'h}}] - f(E + e\phi_2)[R_{\text{P'P'}} + T_{\text{h'P'}}] \}. \quad (10)$$

The further requirement that at equilibrium the charge on the superconductor remains constant yields

$$\begin{aligned} & \int_0^\infty dE \{f(E - e\phi_1)[R_{hP} + T_{h'P}] - f(E + e\phi_1)(R_{Ph} + T_{P'h})\} \\ &= \int_0^\infty dE \{f(E - e\phi_2)[R_{P'h'} + T_{Ph'}] \\ & \quad - f(E + e\phi_2)[R_{h'P'} + T_{h'P'}]\}. \end{aligned} \quad (11)$$

To obtain an expression for the resistance \bar{R} , equations (9) to (11) must be linearized in the voltages. To write down the general result, it is convenient to introduce the quantities $\langle \rho_{ij}^k \rangle = (\langle \rho_{ij} \rangle + \langle \rho_{ki} \rangle)/2$, where angular brackets denote an average value given by

$$\langle \rho_{ij} \rangle = 2 \int_0^\infty dE (-\partial f / \partial E) \rho_{ij}(E). \quad (12)$$

This yields

$$\bar{R} = \frac{\langle R_{PP}^{hh} \rangle + \langle T_{h'P'}^{P'h} \rangle + \bar{\alpha}[\langle R_{h'h'}^{P'P'} \rangle + \langle T_{Ph'}^{h'P} \rangle]}{1 - \langle R_{PP}^{hh} \rangle + \langle R_{hP}^{Ph} \rangle + \bar{\alpha}[\langle T_{hh'}^{PP'} \rangle - \langle T_{Ph'}^{h'P} \rangle]} \quad (13a)$$

where $\bar{\alpha} = (\langle R_{P'h'}^{h'P'} \rangle + \langle T_{Ph'}^{h'P} \rangle) / (\langle R_{Ph}^{hP} \rangle + \langle T_{Ph'}^{h'P} \rangle)$.

For temperatures much lower than the transition temperature T_c † the generalization of equation (8b), obtained by imposing time-reversal symmetry and ignoring small corrections due to the breaking of particle-hole symmetry at $E \neq 0$, is found to be

$$\bar{R} = \frac{\langle R_0 \rangle + \langle T_a \rangle + \delta}{\langle R_a \rangle + \langle T_0 \rangle - \delta} \quad (13b)$$

where $\delta = (\langle R_{hP} \rangle - \langle R_{h'P'} \rangle)^2 / 2(\langle R_{hP} \rangle + \langle R_{h'P'} \rangle + 2\langle T_a \rangle)$ and all other quantities are averaged values of their counterparts in equation (8b).

The generalization to more than one dimension is also achieved through the introduction of suitably defined averages. In this case, for both particles and holes, the leads possess N independent incoming and outgoing channels, corresponding to N discrete transverse k vectors of the incoming or outgoing waves. The 16 elements of $\rho(E)$ are now each replaced by $N \times N$ matrices. The waves in different channels are assumed to be incoherent so that the interference between the separate channels can be neglected. Since the product of the group velocity and density of states for a given channel is channel independent, one finds (cf equation (4)) that at zero temperature, the current in the j th channel is

$$\begin{aligned} I_j = \frac{2e}{h} & \left\{ (\mu_1 - \mu) \left(1 - \sum_{k=1}^N [(R_{PP})_{jk} - (R_{hP})_{jk}] \right) \right. \\ & \left. + (\mu - \mu_2) \left(\sum_{k=1}^N [(T_{hh'})_{jk} - \sum_{k=1}^N (T_{Ph'})_{jk}] \right) \right\}. \end{aligned}$$

† Since the smallest energy scale for variations of the matrix elements ρ_i , is typically $k_B T_c$, terms neglected by ignoring particle-hole asymmetry are of order T/T_c compared to unity.

Hence the total current in the left lead is

$$I = \sum_{j=1}^N I_j = \frac{2eN}{h} \{(\mu_1 - \mu)(1 - \tilde{R}_{PP} + \tilde{R}_{hP}) + (\mu - \mu_2)(\tilde{T}_{hh'} - \tilde{T}_{Ph'})\}$$

where $\tilde{R}_{PP} = N^{-1} \sum_{jk} (R_{PP})_{jk}$ etc.

Similarly, since the inverse group velocity in the j th channel is $(h/2)N_j(0)$, where $N_j(0)$ is the density of states in channel j , the particle density in the j th channel of the left lead is

$$N_j(0) \left[(\mu_1 - \mu) \left\{ 1 + \sum_{k=1}^N [(R_{PP})_{jk} - (R_{hP})_{jk}] \right\} + (\mu - \mu_2) \sum_{k=1}^N [(T_{Ph'})_{jk} - (T_{hh'})_{jk}] \right].$$

Summing over all j and combining the result with a corresponding equation for the right lead, shows that equation (7) generalizes to

$$\begin{aligned} \mu_A - \mu_B &= (\mu_1 - \mu)(1 - \hat{R}_{hP} + \hat{R}_{PP} + \hat{T}_{h'P} - \hat{T}_{P'P})/2 \\ &\quad + (\mu - \mu_2)(1 - \hat{R}_{P'h'} + \hat{R}_{h'h'} + \hat{T}_{Ph'} - \hat{T}_{hh'})/2 \end{aligned}$$

where a $\hat{}$ represents a sum over all channels, weighted by the density of states. For example $\hat{R}_{PP} = [\sum_{jk} N_j(0)(R_{PP})_{jk}]/[\sum_j N_j(0)]$. Furthermore charge conservation yields

$$(\tilde{R}_{P'h'} + \tilde{T}_{Ph'}) (\mu - \mu_2) = (\tilde{R}_{hP} + \tilde{T}_{h'P}) (\mu_1 - \mu). \tag{14}$$

At this stage, it is useful to introduce 4×4 matrices $\hat{\rho}$ and $\tilde{\rho}$, formed by replacing the elements on the right-hand side of equation (3) with their $\hat{}$ or $\tilde{}$ averages. Conservation of quasi-particle probability then yields $\sum_{i=1}^4 \tilde{\rho}_{ij} = 1$, which was used in simplifying equation (14). On the other hand, there is no such condition on $\hat{\rho}$. Consequently, the expression for $\mu_A - \mu_B$ does not simplify and the generalization of equation (8a) takes the more cumbersome form

$$\bar{R} = \frac{1}{2N} \frac{1 - \hat{R}_{hP} + \hat{R}_{PP} + \hat{T}_{h'P} - \hat{T}_{P'P} + \tilde{\alpha}(1 - \hat{R}_{P'h'} + \hat{R}_{h'h'} + \hat{T}_{Ph'} - \hat{T}_{hh'})}{1 - \tilde{R}_{PP} + \tilde{R}_{hP} + \tilde{\alpha}(\tilde{T}_{hh'} - \tilde{T}_{Ph'})} \tag{15a}$$

where $\tilde{\alpha} = (\tilde{R}_{hP} + \tilde{T}_{h'P})/(\tilde{R}_{P'h'} + \tilde{T}_{Ph'})$. In the presence of time reversal and particle-hole symmetry this result takes a form similar to that of equation (8b):

$$\bar{R} = \frac{1}{N} \frac{\tilde{R}_0 + \tilde{T}_a + \delta_1}{\tilde{R}_a + \tilde{T}_0 - \delta_2} \tag{15b}$$

In this expression,

$$\delta_2 = (\tilde{R}_{hP} - \tilde{R}_{h'P'})^2 / 2(\tilde{R}_{hP} + \tilde{R}_{h'P'} + 2\tilde{T}_a)$$

and

$$\delta_1 = \delta_2 + [(\epsilon_P + \epsilon_{h'}) (\tilde{R}_a + \tilde{T}_a) + (\epsilon_P - \epsilon_{h'}) (\tilde{R}_{h'P'} - \tilde{R}_{hP}) / 2] / 2[\tilde{R}_{hP} + \tilde{R}_{h'P'} + 2\tilde{T}_a]$$

where the quantities ϵ_P and $\epsilon_{h'}$, which would vanish if the channel dependence of $N_j(0)$ were ignored, are given by

$$2\epsilon_P = (-\hat{R}_{hP} + \hat{R}_{PP} + \hat{T}_{h'P} - \hat{T}_{P'h}) - (-\tilde{R}_{hP} + \tilde{R}_{PP} + \tilde{T}_{h'P} - \tilde{T}_{P'h})$$

and

$$2\epsilon_{h'} = (-\hat{R}_{P'h'} + \hat{R}_{h'h'} + \hat{T}_{Ph'} - \hat{T}_{h'h'}) - (-\tilde{R}_{P'h'} + \tilde{R}_{h'h'} + \tilde{T}_{Ph'} - \tilde{T}_{h'h'}).$$

This result is readily generalized to finite temperatures. Through a simple extension of the arguments leading to equation (10), the finite temperature version of equation (12b) is again obtained by replacing all reflection and transmission coefficients by their average, defined in equation (12).

The aim of this letter has been to obtain an expression for the boundary resistance of an N-S-N sample, in terms of the reflection and transmission coefficients, which reduces to the Landauer formula in the absence of Andreev scattering. The one-dimensional, zero temperature result is contained in equations (8a) and (8b), with extensions to finite temperatures and higher dimensions contained in equations (13) and (15). The analysis closely follows that of Büttiker *et al* [23], except that in order to maintain a constant quasi-particle charge, an additional self-consistency relationship has been introduced, which fixes the chemical potential of the superconductor. It should be noted that the formulae obtained do not depend on the nature of the superconductivity, except through the implicit assumption that at sufficiently low temperatures, inelastic processes are negligible. For this reason, the results should apply to heavy fermion, high T_c and conventional superconductors. The analysis used to derive these results is rather general. For example, corresponding formulae for the alternative resistance, defined as the ratio of the reservoir potential difference $\mu_1 - \mu_2$ to the current I , are trivially obtained from the equations between (4) and (6) and their multi-channel, finite temperature counterparts. This alternative choice may be relevant to certain measurements on mesoscopic structures, such as hybrid rings [24] and to recent experiments [25] on superconducting-magnetic interfaces. To avoid repetition, the formulae have not been explicitly written down. It should also be noted that equations (8) and the corresponding equations for \tilde{R}_{left} and \tilde{R}_{right} , disagree with a similar result by Blonder *et al* [13], who do not treat the chemical potentials of the leads self-consistently and consequently obtain a result which does not reduce to the Landauer formula in the normal limit.

Historically, our understanding of Anderson localization has benefited greatly from calculations based on the Landauer formula for the resistance of normal disordered solids. As a basis for understanding corrections to the Boltzmann description of quasi-particle transport in inhomogeneous superconductors, one expects the formulae obtained in this letter to play an equally significant role. With a view to investigating the inter-play between diagonal and off-diagonal disorder in the BG equation, calculations extending the work of Hui and Lambert [3, 4] are currently underway and explicit results for \bar{R} based on these formulae will be reported in the near future.

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